

MATH 2D Prep: Differentiation Techniques

1. Use the Chain Rule to find the derivative of $f(x) = \sin^3(2x)$
(Hint: You need to apply Chain Rule twice.)

Solution: By Chain Rule

$$\frac{d}{dx} \sin(2x) = 2 \cos(2x).$$

By Chain Rule again

$$f'(x) = \frac{d}{dx} [\sin(2x)]^3 = 3 \sin^2(2x) \frac{d}{dx} \sin(2x) = 6 \sin^2(2x) \cos(2x)$$

2. If $x = e^t \ln(2t)$, find $\frac{dx}{dt}$

Solution:

by the property of logarithm,

$$\ln(2t) = \ln(2) + \ln(t),$$

So

$$\frac{d}{dt} \ln(2t) = \frac{d}{dt} [\ln(2) + \ln(t)] = \frac{d}{dt} \ln(t) = \frac{1}{t}$$

So

$$\begin{aligned} \frac{dx}{dt} &= e^t \frac{d}{dt} \ln(2t) + \ln(2t) \frac{d}{dt} e^t \\ &= e^t \cdot \frac{1}{t} + e^t \ln(2t) \\ &= e^t \left[\frac{1}{t} + \ln(2t) \right] \end{aligned}$$